ON THE CONSISTENCY OF A REPULSIVE GRAVITY PHASE IN THE EARLY UNIVERSE

A.B. Batista[1], **J. C. Fabris**[2] **and S.V.B. Gonçalves**[3]

Departamento de Física, Universidade Federal do Espírito Santo, 29060-900, Vitória, Espírito Santo, Brazil

Abstract

We exploit the possibility that there is a repulsive gravity phase in the evolution of the Universe. A toy model with a free scalar field minimally coupled to gravity, but with the "wrong sign" for the energy and negative curvature for the spatial section, is studied in detail. The background solution display a bouncing, non-singular Universe. The model is well-behaved with respect to tensor perturbations. But, it exhibits growing models with respect to scalar perturbations whose maximum occurs in the bouncing. Hence, large inhomogeneties are produced. At least for this case, a repulsion phase may destroy homogeneity, and in this sense it may be unstable. A newtonian analogous model is worked out displaying qualitatively the same behaviour. The generality of this result is discussed. We discuss also a quantum version of the classical repulsive phase, through the Wheeler-deWitt equation in mini-superspace, and we show that it displays essentially the same scenario as the corresponding attractive phase.

PACS number(s): 04.20.Cv., 04.20.Me

1 Introduction

Gravity is an attractive interaction. This fact is represented by an universal positive coupling constant. All avaliable observations confirm this statement. However, there is in principle no a priori theoretical reason by which it could be exclude the existence of a repulsive gravity interaction represented by a negative coupling constant or, equivalently, a negative gravitational charge (mass). Indeed, repulsive cosmic interactions has been proposed from time to time in the literature (see for example [1, 2]). The question that comes immediately is if this repulsive cosmic interaction is physically acceptable. To our point of view a specific type of interaction may be considered as non-physical only if it is mathematical inconsistent and/or it leads to some undesirable features like some kind of unstability or divergences in physical quantitities.

[1]e-mail: brasil@cce.ufes.br
[2]e-mail: fabris@cce.ufes.br
[3]e-mail: svbg@if.uff.br

The aim of this work is to verify to which extent a repulsive gravity may be excluded as non-physical. In [1] repulsive interactions in the primordial Universe appears as consequence of coupling fermionic and electromagnetic field with gravity, which is coupled conformally to a scalar field. The couplings determine the repulsive cosmic phase, which becomes attractive in a latter phase, with a changing of sign of the gravitational coupling. In [2], the repulsive gravity appears as consequence of the employement of conformal gravity, through the Weyl's tensor: the conformal coupling to with matter fields leads to a repulsive gravity at cosmological level, keeping its attractive character at local level. The curvature of the spatial section is necessarily negative. It has also been argued that the recent results concerning an accelerated expansion of the Universe [3, 4] are due to a cosmic repulsion effect [5]. In fact, such repulsive effect appears for example in the Brans-Dicke dust cosmological solution [6, 7] for negative values of ω, although in this case it is not clear how to reconcile this "cosmic" results with the local tests of gravity.

In fact, repulsive effects are frequently explicitly or implicitly refered to mainly when comes to scene situations where a singularity is presented, like in the big-bang scenario. In what concerns the primordial Universe, some kind of repulsive effect is necessary if one wants to avoid the initial singularity. In principle, the violation of the strong energy condition seems enough to do the job. But, we can think, in a more general level, if repulsive gravity violating all energy conditions [8] can exist and to become the dominant interaction at some moment, avoiding the appearence of a singularity. This could be the case if there is some repulsive gravity effect, represented by an universal negative coupling constant, which dominates in extreme situations as it occurs in the very early Universe and becoming negligeble latter.

We intend to verify if such repulsive phase may lead to coherent cosmological scenario. To perform this study, we will require from this possible repulsive phase to be mathematically consistent and not to contain any anomaly either at classical and quantum level. To be explicit, we will work out a toy model consisting of Einstein's equations coupled minimally to a free scalar field. The possibility that this scalar field appears in Einstein's equation with a wrong sign will be exploited. It will be shown that a consistent and singularity free background cosmological scenario comes out if the curvature of the spatial section is negative. In some sense, this "exotic" scalar field ameliorates things with respect to the "normal" scalar field, which leads to a singular cosmological scenario. But, a perturbative analysis will reveal the presence of some kind of instabilities near the bounce, exactly where the repulsive phase becomes dominant: the Universe may become too inhomogenous there, and the initial hypothesis of isotropy and homogeneity may not survive during the bouncing. This results are somehow supported by a similar newtonian analysis.

Later, we will verify if the some more anomaly of this "exotic" field can appear in its quantum behaviour. This is a very difficult question to answer, since there is not yet a full consistent quantum gravity theory and the study of quantum effects in a fixed background, what could be another possibility, represents a very stringent simplification of our dynamical model. However, we try to obtain some hints about a posssible answer to that question studying the Wheeler-deWitt equation for that model in mini-superspace. In this case, the equations are exactly solvable. We verify that the solutions with negative and positive energy at this quantum level are essentially the same, and no further restriction for a repulsive gravity phase can be obtained through this way.

In next section we describe our toy model, determining the background solutions. In section 3 a perturbative analysis of the previously founded solution is carried out. In section 4 the Wheeler-deWitt equation in mini-superspace is solved both for the "normal" and "exotic" scalar field, and the results are compared. In section 5 the conclusions are presented.

2 A scalar field model

Let us consider gravity coupled to a scalar field, but with no restriction to the sign of this coupling. Hence, in our conventions [4], the Lagrangian reads

$$L = \sqrt{-g}\Big\{R \pm \phi_{;\rho}\phi^{;\rho}\Big\} \tag{1}$$

where the positive (negative) sign inside the brackets implies negative (positive) energy density, hence repulsive (attractive) gravitational effects. From this Lagrangian, it follows the field equations,

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \mp\Big\{\phi_{;\mu}\phi_{;\nu} - \frac{1}{2}g_{\mu\nu}\phi_{;\rho}\phi^{;\rho}\Big\} \quad , \tag{2}$$

$$\Box\phi = 0 \tag{3}$$

The geometry is described by the Friedmann-Robertson-Walker metric,

$$ds^2 = dt^2 - a^2(t)\Big[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\Big] \tag{4}$$

where $k = 0, 1, -1$ depending if the spatial section is flat, closed or opened. Since we will be interested in a homogenous and isotropic Universe, the scalar field ϕ must be function only of the cosmic time, $\phi = \phi(t)$.

Inserting the metric (4) into the field equations (2,3), we obtain the equations of motion,

$$3\Big(\frac{\dot a}{a}\Big)^2 + 3\frac{k}{a^2} = \mp\frac{1}{2}\Big(\frac{\dot\phi}{\phi}\Big)^2 \quad , \tag{5}$$

$$\ddot\phi + 3\frac{\dot a}{a}\dot\phi = 0 \quad . \tag{6}$$

When we choose the upper sign in (5) there is no real solution for $a(t)$ unless $k = -1$. Hence, we will consider just the open Universe. From (6), we obtain $\phi = \phi_0 a^{-3}$. Inserting this first integral in (5), and changing to the conformal time $dt = a(\eta)d\eta$, we find the following solutions:

[4] The conventions are $R_{\mu\nu}\partial_\rho\Gamma^\rho{}_{\mu\nu} - \partial_\nu\Gamma^\rho{}_{\mu\rho} + ...$, sign$g = (+ - --)$.

- Upper sign:

$$a(\eta) = a_0\sqrt{\cosh 2\eta} \quad , \tag{7}$$
$$\phi(\eta) = \phi_0 \arctan\Big[(\cosh\eta \pm \sinh\eta)^2\Big] \quad ; \tag{8}$$

- Lower sign:

$$a(\eta) = a_0\sqrt{\sinh 2\eta} \quad , \tag{9}$$
$$\phi(\eta) = \phi_0 \ln\tanh\eta \quad . \tag{10}$$

The solutions corresponding to a repulsive gravity phase (upper sign) describe an Universe with a bounce which is non-singular. The positive energy solutions (lower sign) describe an Universe with an initial singularity. Hence, from the point of view of the background, the negative energy solution seems more interesting than the positive energy solution, due to the existence of an initial singularity in the latter. However, we can ask if such bounce, eternal, singularity-free Universe is not plagued with stability problems. This is the subject of the next section.

Before turn to this perturbative analysis, one may ask if the existence of a repulsive phase is restricted to the open geometry. In fact, if we consider just one field, this is true. However, in mixed models, where there is a fluid that acts attractivelly and other repulsivelly is it possible to have solutions even if the curvature is positive or zero. Let us consider for example a radiative fluid whose coupling is positive and a stiff matter fluid (in fact, a free scalar field to which it is equivalent) whose coupling is negative, and which interacts only through the geometry. The equations of motion are reduced essentially to

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{k_1}{a^4} - \frac{k_2}{a^6} \quad , \tag{11}$$

where k_1 and k_2 are constants. The spatial curvature is zero. The solution for this equation, in terms of the conformal time, is

$$a = a_0\sqrt{\eta^2 + C^2} \quad , \tag{12}$$

where C is a constant. This describe a bounce Universe, which goes asymptotically to a radiation dominated Universe. It is curious that if the roles of these fields where interchanged, leading to a repulsive radiation field and an attractive stiff matter field, the solution should be written as

$$a = a_0\sqrt{C^2 - \eta^2} \quad . \tag{13}$$

This is a singular Universe which can exist only for a finite period of time.

3 Evolution of tensor and scalar perturbations

We will now perturb the background described before for the scalar field model. Hence, we writte the metric and the scalar field as $g_{\mu\nu} = \overset{0}{g}_{\mu\nu} + h_{\mu\nu}$ and $\phi = \overset{0}{\phi} + \delta\phi$, where $\overset{0}{g}_{\mu\nu}$ and

$\overset{0}{\phi}$ are the background solutions founded previously and $h_{\mu\nu}$ and $\delta\phi$ are small perturbations around them. Due to the coordinate transformation invariance of the field equations, we can impose a coordinate condition. We choose the synchrounous coordinate condition $h_{\mu 0} = 0$; the only non-negative components will be those such that $\mu = i$ and $\nu = j$. The perturbations can be decomposed in purelly tensor perturbations, described by a tracelless transverse components of $h_{\mu\nu}$, vectorial perturbations and scalar perturbations [9]. The vectorial perturbations are quite trivial, leading to terms that are proportional to the scale factor. Tensor and scalar perturbations are less trivial, and we study them separately.

In order to perform this perturbative analysis, we rewritten the field equations as

$$\begin{aligned} R_{\mu\nu} &= \mp\phi_{;\mu}\phi_{;\nu} \quad , & (14)\\ \Box\phi &= 0 \quad . & (15)\end{aligned}$$

3.1 Tensor perturbations

Perturbing (14), retaining only the transverse tracelless component, the metric perturbations are described by a single equation, which reads, in terms of conformal time,

$$h'' - 2\frac{a'}{a}h' + \biggl\{\tilde{n}^2 - 2\biggl(\frac{a''}{a} - \frac{a'^2}{a^2}\biggr)\biggr\}h = 0 \quad . \qquad (16)$$

To obtain this equation the metric perturbation has been written as $h(\eta,\vec{x})_{ij} = h(\eta)Q_{ij}(\vec{x})$, such that $Q_{kk} = Q_{ik|k} = 0$ and $\nabla^2 Q_{ij} = -n^2 Q_{ij}$. Hence, the Q_j are eigenfunctions of the laplacian operator defined in the constant curvature spatial hypersurface. Moreover $\tilde{n}^2 = n^2 + 2k = n^2 - 2$, due to the fact that the curvature is negative.

We must insert in (16) the solutions (7) and (9). For the first case, which corresponds to the repulsive scalar field, we find the equation

$$h'' - 2\tanh 2\eta h' + \biggl\{\tilde{n}^2 - \frac{4}{\cosh^2 2\eta}\biggr\}h = 0 \quad . \qquad (17)$$

This equation may be solved through the transformations $h(\eta) = \cosh 2\eta\lambda(\eta)$ and $x = \sinh 2\eta$ by which it takes the form

$$(1 + x^2)\ddot{\lambda} + 2x\dot{\lambda} + \frac{\tilde{n}^2}{4}\lambda = 0 \quad , \qquad (18)$$

where the dot means derivative with respect to x. This equation admits a solution under the form of hypergeometric functions, leading to the final expressions

$$\begin{aligned} h_1 &= \cosh 2\eta\, {}_2F_1\biggl(\frac{1}{4}(1 + \sqrt{1-\tilde{n}^2}), \frac{1}{4}(1 - \sqrt{1-\tilde{n}^2}), \frac{1}{2}, -\sinh^2\eta\biggr) \quad , & (19)\\ h_2 &= \sinh 2\eta\cosh 2\eta\, {}_2F_1\biggl(\frac{1}{4}(3 + \sqrt{1-\tilde{n}^2}), \frac{1}{4}(3 - \sqrt{1-\tilde{n}^2}), \frac{3}{2}, -\sinh^2\eta\biggr) & (20)\end{aligned}$$

From the previous definitions we have $-2 \leq \tilde{n}^2 < \infty$.

These solutions for the tensor modes exhibit a quite regular behaviour. The perturbations tends to diverge at both asymptotics, where anyway the scale factor goes to infinity, but they are strongly supressed as they approach the bounce. The general features are sketched in figures 1 and 2. Hence, the background solutions are stable against tensor perturbations: the production of gravitational waves does not destroy the configuration. This result will be discussed later.

Repeating the same analysis for the postive energy solution, we find the following expressions for the tensor perturbations for this case:

$$h_3 = \sinh 2\eta \,{}_2F_1\biggl(\frac{1}{4}(1+\sqrt{1-\tilde{n}^2}),\frac{1}{4}(1-\sqrt{1-\tilde{n}^2}),\frac{1}{2},\cosh^2\eta\biggr) \quad , \tag{21}$$

$$h_4 = \sinh 2\eta \cosh 2\eta \,{}_2F_1\biggl(\frac{1}{4}(3+\sqrt{1-\tilde{n}^2}),\frac{1}{4}(3-\sqrt{1-\tilde{n}^2}),\frac{3}{2},\cosh^2\eta\biggr) \tag{22}$$

The behaviour of these. solutions are displayed in figures 3 and 4. They have essentially the same features of the previous case, but "cutting" the half of the graph. No anomalous behaviour occurs, and all perturbations emerge from a zero value from the singularity exhibiting a growing or oscillatory behaviour, depending on the value of n at the infinity asymptotic.

3.2 Scalar perturbations

Now, we turn to the scalar modes. We decompose the metric as $h_{ij}(\eta,\vec{x}) = h_{ij}(\eta)Q(\vec{x})$, where $\nabla^2 Q = -n^2 Q$. Perturbing now the equations (14,15), we obtain the following equations:

$$h'' + \frac{a'}{a}h' = \mp 4\phi'\delta\phi' \quad , \tag{23}$$

$$\delta\phi'' + 2\frac{a'}{a}\delta\phi' + n^2\delta\phi = \frac{1}{2}\phi' h' \quad , \tag{24}$$

where $h = \frac{h_{kk}}{a^2}$. From (24), we can express h' and h'' in terms of $\delta\phi$ and its derivatives. We obtain a third order differential equation:

$$\delta\phi''' + \biggl(3\frac{a'}{a} - \frac{\phi''}{\phi'}\biggr)\delta\phi' + \biggl(n^2 + 2\frac{a''}{a} - 12\frac{a'^2}{a^2} + 12 - 2\frac{a'}{a}\frac{\phi''}{\phi'}\biggr)\delta\phi' + \biggl(\frac{a'}{a} - \frac{\phi''}{\phi'}\biggr)n^2\delta\phi = 0 \quad . \tag{25}$$

The fact that we end up with a third order equation is a consequence of the coordinate condition chosen: the synchronous coordinate condition contains a residual coordinate freedom, as it is well known [10]. This enables us to reduce the order of the equation, since we know the effect on $\delta\phi$ of this residual coordinate freedom.

First we will consider the negative energy case. Inserting the background solutions and defining the transformation

$$\delta\phi = \cosh^{-3/2} 2\eta\lambda \quad , \tag{26}$$

we obtain the equation

$$\lambda''' - 4\tanh 2\eta\lambda'' + \biggl(n^2 + 5\tanh^2 2\eta - 2\biggr)\lambda' = 0 \tag{27}$$

which can be solved through the same kind of transformation as in the case of the tensor perturbations. The final solutions are:

$$\begin{aligned}\delta\phi_1 &= \cosh^{-3/2} 2\eta\Big[C_1\int \cosh^{1/2} 2\eta \times \\ &\quad {}_2F_1\Big(-1/4(1+\sqrt{1-n^2}), -\frac{1}{4}(1-\sqrt{1-n^2}), \frac{1}{2}, -\sinh^2 2\eta\Big)+C_2\Big] \quad , \quad (28)\\ \delta\phi_2 &= \cosh^{-3/2} 2\eta\Big[C_3\int \cosh^{1/2} \eta \sinh 2\eta \times \\ &\quad {}_2F_1\Big(-1/4(1+\sqrt{3-n^2}), -\frac{1}{4}(3-\sqrt{1-n^2}), \frac{3}{2}, -\sinh^2 2\eta\Big)+C_4\Big] \quad . \quad (29)\end{aligned}$$

The C_i are integration constants.

For the positive energy case, the procedure is essentially the same, and the final expressions are

$$\begin{aligned}\delta\phi_3 &= \sinh^{-3/2} 2\eta\Big[D_1\int \sinh^{1/2} \times \\ &\quad {}_2F_1\Big(-1/4(1+\sqrt{1-n^2}), -\frac{1}{4}(1-\sqrt{1-n^2}), \frac{1}{2}, \cosh^2 2\eta\Big)+D_2\Big] \quad , \quad (30)\\ \delta\phi_4 &= \sinh^{-3/2} 2\eta\Big[D_3\int \cosh 2\eta \sinh^{1/2} 2\eta \times \\ &\quad {}_2F_1\Big(-1/4(1+\sqrt{3-n^2}), -\frac{1}{4}(3-\sqrt{1-n^2}), \frac{3}{2}, \cosh^2 2\eta\Big)+D_4\Big] \quad . \quad (31)\end{aligned}$$

Again, the D_i are integration constants.

The analysis of the results obtained above are more involved. In principle we are tempted to say that the perturbations are regular, since the integrand are regular. However, they diverge at infinity which is not a serious divergence. But, for the case of repulsive coupling, the integration leads to very large values of the perturbations near the bounce. The earlier the perturbations are originated, the larger are their values near the bounce. For perturbations originated in the first asymptotic, their amplitude in the bouncing tends to a divergent value. That means that the Universe may become highly inhomogenous near the bounce, or even unstable if we allow the perturbations to orginate at $\eta \rightarrow -\infty$, and the initial configuration is destroyed. This results has been confirmed by direct numerical integration. On the other hand, the positive energy solutions display a more regular behaviour throughout the evolution of the Universe, as it is shown in figure 6.

It could be argued that the relevant quantity is not $\delta\phi$, but the associated density contrast, $\Delta_\phi = \frac{\delta\phi'}{\phi'}$. However, for the negative energy case, this quantity takes very large values also near the bounce, exhibiting essentially the same behaviour as $\delta\phi$.

3.3 A newtonian analysis

This result can be confirmed qualitatively through a newtonian model. This model is described by the following equations [9]:

$$\dot{\rho} + \nabla.(\rho\vec{v}) = 0 \quad , \qquad (32)$$

$$\dot{\vec{v}} + \vec{v}.\nabla(\vec{v}) = -\frac{\nabla p}{\rho} + \vec{g} \quad , \tag{33}$$
$$\nabla.\vec{g} = 4\pi G\rho \quad , \tag{34}$$

where we have consider the wrong sign associated with the gravitational coupling. These equations admit solutions for a isotropic and homogenous Universe such that

$$\rho = \frac{\rho_0}{a^3} \quad , \quad \vec{v} = \vec{r}\frac{\dot{a}}{a} \quad , \quad \vec{g} = \vec{r}\frac{4\pi G\rho}{3} \quad , \tag{35}$$

where the function a obeys the equations

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = -\frac{8\pi G\rho}{3} \quad , \quad \frac{\ddot{a}}{a} = \frac{4\pi G\rho}{3} \quad . \tag{36}$$

There is a solution if $k < 0$. In this case, after passing to the conformal time, the solution is

$$a(\eta) = a_0(1 + \cosh\eta) \quad , \tag{37}$$

which describe a bounce Universe also.

Perturbing the equations (32,33,34), we can find out an equation for the density contrast $\Delta = \frac{\delta\rho}{\rho}$, which reads in terms of the conformal time:

$$\Delta'' + \frac{a'}{a}\Delta' + \left\{n^2 v_s^2 + 3\left(\frac{a''}{a} - \frac{a'^2}{a^2}\right)\right\}\Delta = 0 \tag{38}$$

where n denotes as before the wavenumber of the perturbation. This equation differs from the attractive gravity case by the form of the function $a(\eta)$ and by the sign of the last term in (38). There is analytical solutions for (38) in the long wavelength limit ($n \rightarrow 0$):

$$\Delta_+ = E_1\left(-\frac{3\eta\sinh\eta}{(1+\cosh\eta)^2} + \frac{5-\cosh\eta}{1+\cosh\eta}\right) \quad ; \tag{39}$$
$$\Delta_- = E_2\frac{\sinh\eta}{(1+\cosh\eta)^2} \quad . \tag{40}$$

E_1 and E_2 are integration constants. It is easily to see that the "growing mode" represented by the solution Δ_+ has a maximum near the bouncing. Qualitatively, the relavistic result is reproduced. However, the maximum in the newtonian case is not so important as the relativistic case. This seems to be due to the contribution of the pressure to the effective mass in the relativistic situation, increasing the repulsive effect.

4 A quantum cosmological analysis

One may ask if some anomalies may also be present at quantum level. A complete answer to this problem seems very difficult to obtain. We could study the presence of quantum fields in the background described before; but since is complete regular, we may guess that no anomolous behaviour can be expected from this approach. Or we may turn to the Wheeler-dewitt equation [11, 12]. In this case, the first simplification we are, in some sense, obliged to make is to freeze out all degrees of freedoms except those related to the scale factor and the scalar field, working in the so-called mini-superspace. Perhaps, we could take more degrees of freedom into account, but we do not think that the final results would be modified drastically.

Hence, we begin from the Lagrangian (1). Inserting in it the metric (4) and the time-dependence of the scalar field, we end up, after integration by parts, with the expression

$$L = \biggl\{6\frac{a\dot{a}^2}{\sqrt{N}} - 6ka\sqrt{N} \pm \frac{a^3\dot{\phi}^2}{\sqrt{N}}\biggr\} \quad , \tag{41}$$

where N is the lapse function connected we the time reparametrization freedom. From it, the conjugate momenta are obtained, through the relation $\phi_q = \frac{\partial L}{\partial \dot{q}}$:

$$\pi_a = 12\frac{a\dot{a}}{\sqrt{N}} \quad , \quad \pi_\phi = \pm 2\frac{a^3\dot{\phi}}{\sqrt{N}} \quad . \tag{42}$$

Hence, the hamiltonian can be constructed canonically:

$$H\sqrt{N}\biggl\{\frac{1}{24}\frac{\pi_a^2}{a} \pm \frac{1}{4}\frac{\pi_\phi^2}{a^3} + 6ka\biggr\} \quad . \tag{43}$$

We must remark that in the derivation of the above hamiltonian, we have discarded a surface term. Hence, the manifold must be compact. It is possible to have compact manifolds even if the spatial curvature is zero or negative, as we want to consider here.

The canonical quantization is now applied. The hamiltonian is taken as an operator which acts over a wavefunction. The substitutions $\pi_a \rightarrow -i\hbar\frac{\partial}{\partial_a}$ and $\pi_\phi \rightarrow -i\hbar\frac{\partial}{\partial_\phi}$ are included. Then, the Wheeler-deWitt equation in the mini-superspace is obtained (with $\hbar = 1$):

$$\biggl\{\partial_a^2 + \frac{p}{a}\partial_a \pm \frac{6}{a^2}\biggl[\partial_\phi^2 + \frac{q}{\phi}\biggr] - 144ka^2\biggr\}\Psi = 0 \quad . \tag{44}$$

The parameters p and q were introduced in order to take into account the order ambiguity. The upper (lower) sign in the second term refer to a classical negative (positive) energy. The Wheeler-deWitt equation may be solved by the separation of variable method. Hence, we writte $\Psi(a,\phi) = \alpha(a)\beta(\phi)$. Taking $k = -1$, two equations are obtained:

$$\alpha'' + \frac{p}{a}\alpha' + \biggl\{144a^2 \pm 6\frac{c}{a^2}\biggr\}\alpha = 0 \quad , \tag{45}$$

$$\beta'' + \frac{q}{\phi} - c\beta = 0 \quad , \tag{46}$$

where c is a constant connected to the separation of variables. An important aspect is that the possibility of having negative (positive) energy implies to choose the upper (lower) sign in (45). Since c may have any value in the complex plan, the choice of one possibility or another changes nothing in the final analysis. According to this simplified quantum model, positive or negative classical energies leads essentially to the same quantum model.

In order to be complete, we writte down the solutions for equations (45,46). They read,

$$\alpha = a^{(1-p)/2}\Big\{A_1 J_\nu(\sqrt{36}a^2) + A_2 J_{-\nu}(\sqrt{36}a^2)\Big\} \quad , \tag{47}$$

$$\beta = \phi^{(1-q)/2}\Big\{B_1 I_r(c\phi) + B_2 K_r(c\phi)\Big\} \quad , \tag{48}$$

where J, I and K are Bessel's function and modified Bessel's function, $\nu = \frac{1}{4}\sqrt{1-p\pm 6c}$, $r = \frac{1}{2}(1-q)$. A_i and B_i are integration constants.

These solutions, in a slightly different form, were analyzed in [13, 14]. In general, they predict a singular Universe. However, gaussian superposition of these solutions may lead to non-singular univers [14]. Hence, both classical cases typical of positive or negative energies are covered by these solutions of the Wheeler-deWitt equation.

5 Final comments

In this paper, we worked out a cosmological model including a repulsive phase verifying if it is consistent physically and mathematically. In order to be specific, a model containing a minimal coupling between gravity and a free scalar field has been studied. A repulsive phase is obtained when the energy of this scalar field is negative. Mathematically, consistent solutions can be obtained only if the spatial curvature is negative. This fact has already been remarked by [2] in the context of a conformal gravity. We argued that in fact repulsive effects may appear in cosmology only when mixed with some other attractive fluid. From the physical point of view, we try to verify if this solution is stable and if it presents any anomaly at the level of the Wheeler-deWitt equation in the minisuperspace.

The first feature to be noted is that while the positive energy is singular, the negative energy presents a bounce, being non singular. Surprisingly, this seems to favor the negative energy solution. We turn then to a perturbative analysis for both cases. Scalar and tensor perturbations were studied. Tensor perturbations present a very regular behaviour, diverging only in the asymptotical limit where the scale factor also goes to infinity. This is not dangerous of course. But, scalar perturbations reveal exactly the opposite behaviour: they are regular in the asymptotics, but assume extremelly large values near the bounce, where the scale factor reachs its minimum value. At this moment, the repulsive effects are the dominant one, and even if they lead to nice features for the background, they carry also the seed of their destruction: the Universe becomes too inhomogenous. This results has been qualitatively confirmed through a newtonian analysis.

These different results for tensor and scalar perturbations are not so surprising. Tensor perturbations are sensitive essentially to the scale factor behaviour and not directly to what fluid content leads to this behaviour. The regularity observed here seems just to indicate that it is possible to have stable bounce scenario, which can be obtained with less extreme hypothesis like the violation of the strong energy condition only. On the other hand, scalar perturbations are sensitive to the matter content and couplings of the model. Hence, they feel directly the repulsive character of the scalar field introduced here: the repulsive field may lead to a large amplification of perturbations in a finite time, when the scale factor takes its minimum value, from where the destruction of the homogeneity. In this sense, the model is unstable.

The quantum model developed here, with a minisuperspace formulation of the Wheeler-deWitt equation, had not revealed any anomaly in the negative energy model. In fact, the solutions with positive energy are indistinguishable of those with negative energy. Moreover, previous analysis made of the positive energy model [13, 14] showed the existence of singular cosmological models and non-singular models. Hence, both possibilities corresponding to the classical solutions are covered.

The main conclusion of this work is that a repulsive phase may lead to a non-singular Universe but which is unstable in the sense that the initial hypothesis, like homogeneity, can not survive the repulsive era. One may ask how general is this result. Of course, we have studied a very specific model. But, this model is somehow similar, from the point of view of the behaviour of the scale factor, to that exposed in [2]; hence, we can argue that the cosmic repulsion coming from the conformal gravity may also be unstable, in the sense employed here, even if the study of this question for the conformal gravity theory is much more involved due to the complexity of field content. On the other hand, the model presented in [1] indicates a changing of the sign of the cosmological constant, and it has already been shown in other situations that such transition from anti-gravity to gravity phase leads to instabilities [15, 16]. Hence, even if a deeper and more general study is needed, all this results suggests the instability of a cosmic repulsion phase.

References

[1] M. Novello, Phys. Lett. **90A**, 347(1982);

[2] P.D. Mannheim, *Attractive and repulsive gravity*, gr-qc/0001011;

[3] S. Perlmutter et al., Nature, **391**, 51(1998);

[4] A.G. Riess et al., Astrophys. J. **116**, 1009(1998);

[5] P.D. Mannheim, *Implications of cosmic repulsion for gravitational theory*, astro-ph/9804335;

[6] A.B. Batista, J.C. Fabris and R. de Sá Ribeiro, gr-qc/0001055;

[7] L.E. Gurevich, A.M. Finkelstein and V.A. Ruban, Astrophys. Spac. Sci. **22**, 231(1973);

[8] M. Visser, *Lorentzian wormholes*, AIP Press, New York (1995);

[9] S. Weinberg, **Gravitation and cosmology**, Wiley, New York (1972);

[10] P.J.E. Peebles, *The large scale structure of the Universe*, Princeton University Press (1980);

[11] J.A. Wheeler, in *Battelle Rencontres: 1967 Lectures in Mathematical Physics*, ed. by B. DeWitt and J.A.Wheeler (Benjamin New York, 1968); B. S. DeWitt, Phys. Rev. **160**, 1113 (1967);

[12] N. Pinto-Neto, Quantum Cosmology in *Cosmology and Gravitation II*, ed. by M. Novello (Editions Frontières, Gyf-sur-Yvette, 1996);

[13] R. Colistete Jr., J.C. Fabris and N. Pinto Neto, Phys. Rev. **D56**, 4707(1998);

[14] R. Colistete Jr., J.C. Fabris and N. Pinto Neto, *Gaussian superpositions in scalar-tensor quantum cosmological model*, to appear in Physical Review D, gr-qc/0005013;

[15] A.A. Starobinskii, Pis'ma Astron. Zh. **7**, 67(1981); Sov. Aston. Lett. **7**, 36(1981);

[16] R. Balbinot, J.C. Fabris and R. Kerner, Phys. Rev. **D42**, 1023(1990).

Figure captions

Figure 1: Behaviour of $h_1(\eta)$ for $n = 1$.

Figure 2: Behaviour of $h_2(\eta)$ for $n = 1$.

Figure 3: Behaviour of $h_3(\eta)$ for $n = 1$.

Figure 4: Behaviour of $h_4(\eta)$ for $n = 1$.

Figure 5: Behaviour of $\delta\phi(\eta)$ $(n = 0.5)$ for the negative energy solutions.

Figure 6: Behaviour of $\delta\phi(\eta)$ for $(n = 0.5)$ for the positive energy solutions.